

# A Tutorial in Connectome Analysis (I): Topological and Spatial Features of Brain Networks

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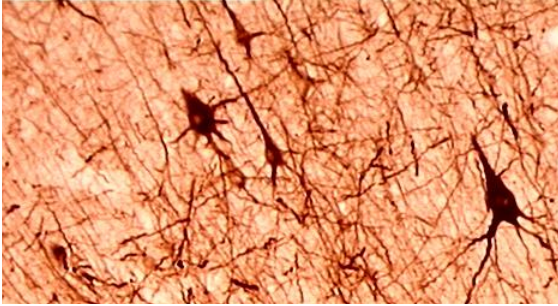
<http://www.biological-networks.org>

# Outline

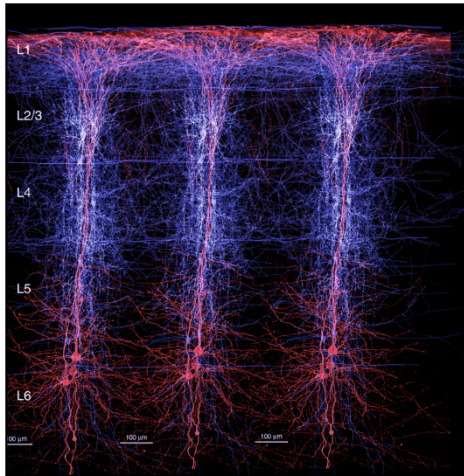
- What are neural networks?
- Introduction to network analysis
- How can the fibre tract network structure be examined?
- Topological network organisation

# What are neural networks?

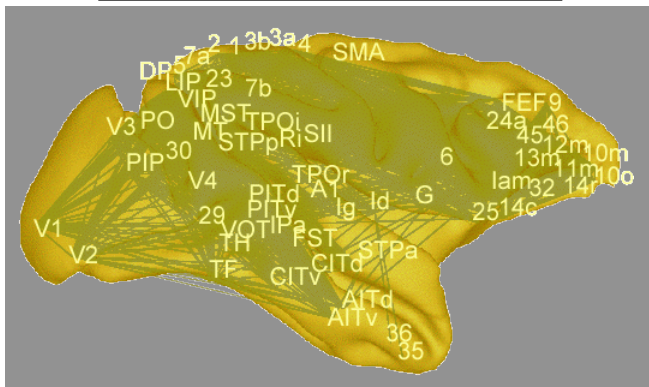
# Levels of connectivity



# Axons between neurons



## Links between cortical columns

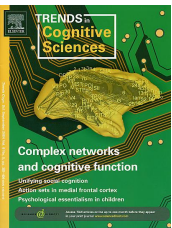


## Fibre tracts between brain areas

# Types of connectivity

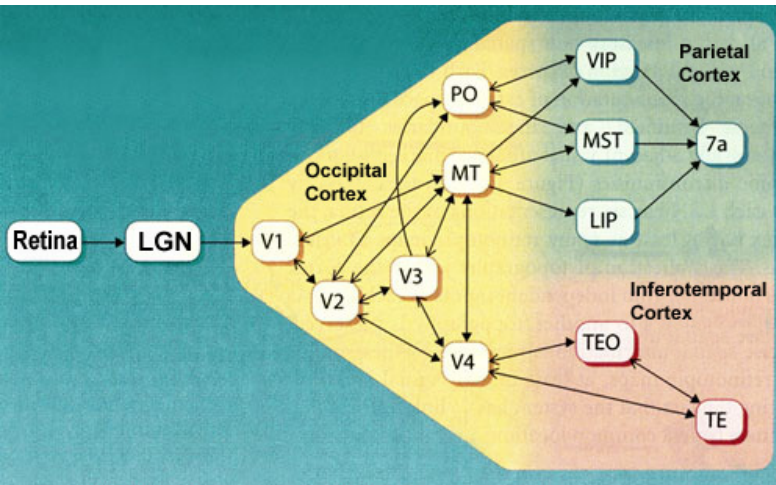


- Structural / Anatomical (connection):  
two regions are connected by a fibre tract
- Functional (correlation):  
two regions are active at the same time
- Effective (causation):  
region A modulates activity in region B

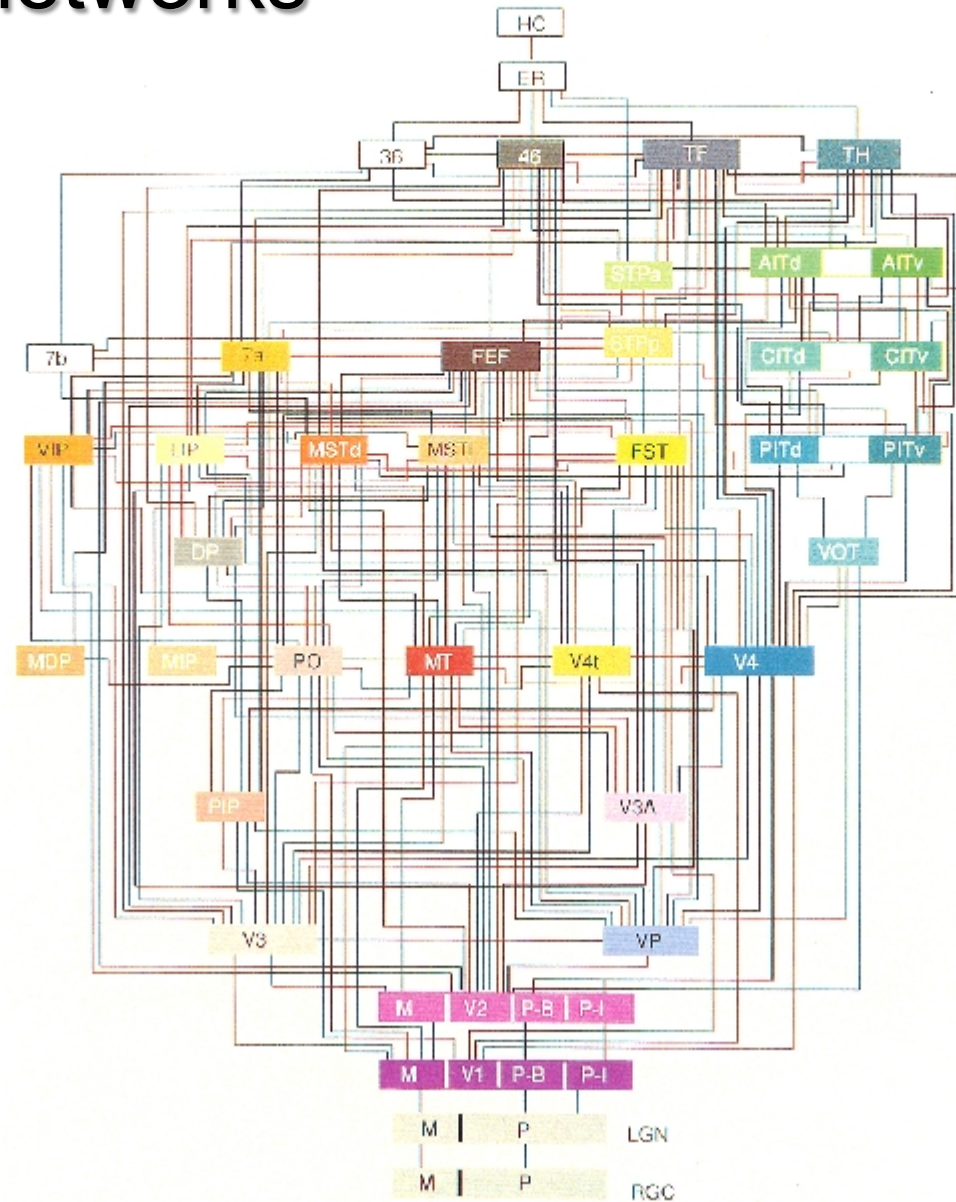


Sporns, Chialvo, Kaiser, Hilgetag. Trends in Cognitive Sciences, 2004

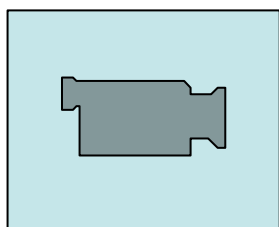
# Cortical networks



**Dorsal and ventral  
visual pathway**



**Visual system**



# Introduction to network analysis



# Network Science

Rapidly expanding field:

Watts & Strogatz, *Nature* (June 1998) cited 4,000+ times

Barabasi & Albert, *Science* (October 1999) cited 4,000+ times

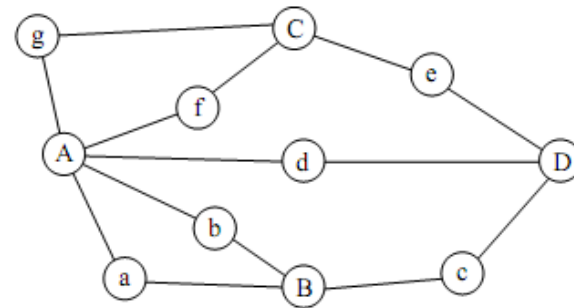
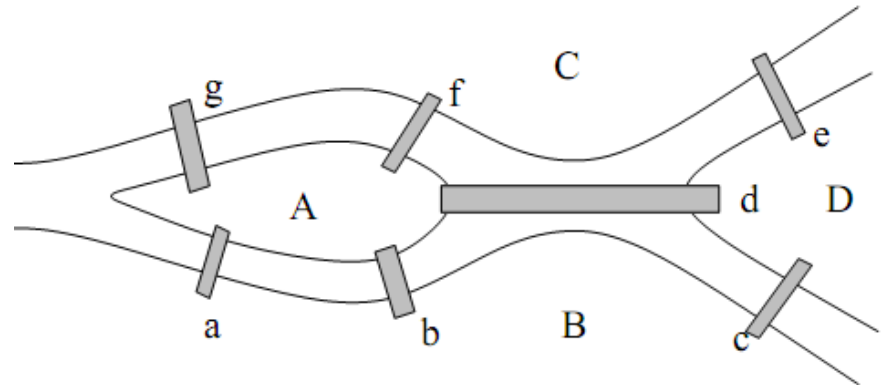
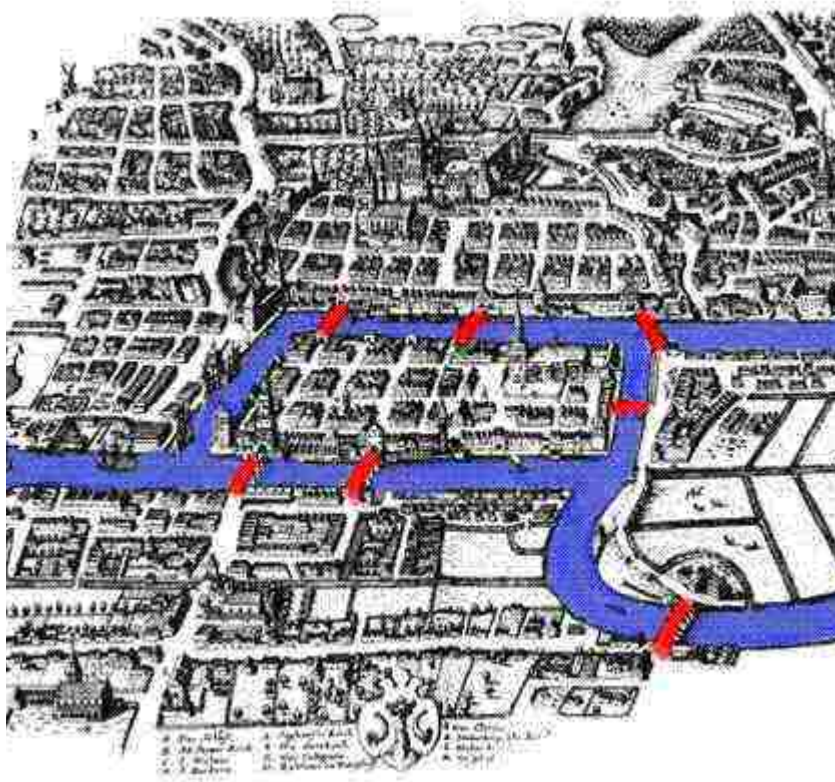
Modelling of SARS spreading over the airline network  
(Hufnagel, *PNAS*, 2004)

Identity and Search in Social Networks  
(Watts et al., *Science*, 2002)

The Large-Scale Organization of Metabolic Networks.  
(Jeong et al., *Nature*, 2000)

First textbook on brain connectivity  
(Sporns, 'Networks of the Brain', MIT Press, October 2010)

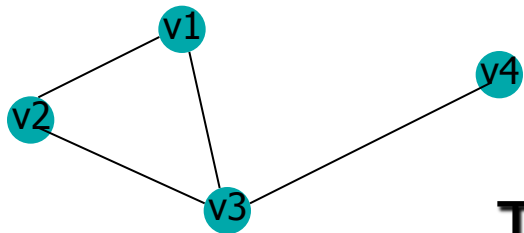
# Origin of graph theory: Leonhard Euler, 1736



Bridges over the river Pregel in Königsberg (now Kaliningrad)  
Euler tour: path that visits each edge and returns to the origin

# Graphs

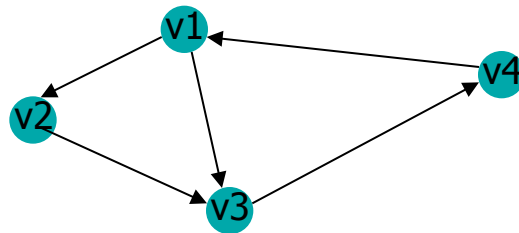
- Graph: set of nodes and edges (non-directed)  
 $G = (V, E)$
- Set of nodes:  $V$  (singular: vertex; plural: vertices)
- Set of edges:  $E \subseteq V \times V$
- E.g.,  $V = \{v1, v2, v3, v4\}$ ,  
 $E = \{(v1, v2), (v1, v3), (v2, v3), (v3, v4)\}$



**Topology**

# Directed graphs (Digraphs)

- Graph: set of nodes and *arcs* (directed)
- Set of nodes (vertices):  $V$
- Set of edges:  $E \subseteq V \times V$ , the order matters
- E.g.,  $V = \{v_1, v_2, v_3, v_4\}$ ,  
 $E = \{(v_1, v_2), (v_1, v_3), (v_2, v_3), (v_3, v_4), (v_4, v_1)\}$



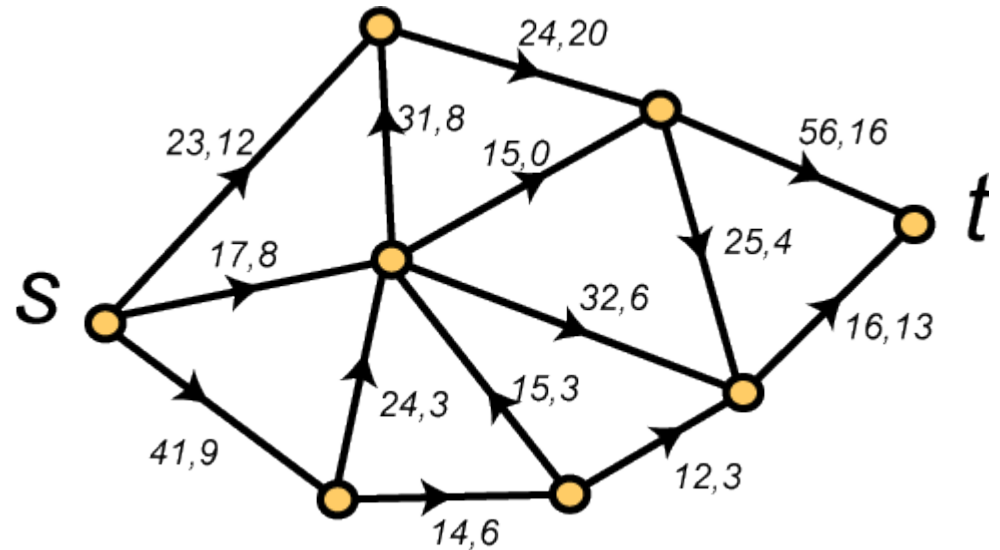
# Graphs and Networks

In theory (mathematics)

Graph:  $G=(V,E)$

*Network*:  $N=(G, s, t, c)$   
defined by graph  $G$  with  
source  $s$ , sink  $t$ , and  
edge capacity  $c$

(examples: electricity/power  
grid, water flow, metabolic flux)



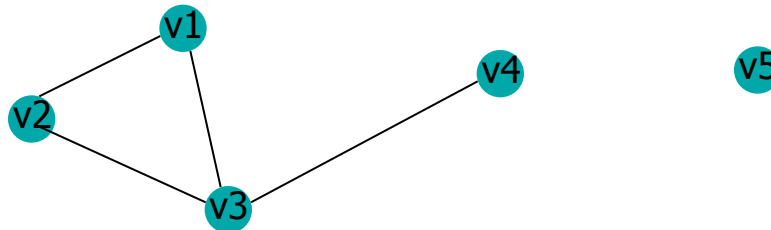
In reality (CS, engineering, economics, life and social sciences):  
term network used throughout (as in this course)

# Nodes in graphs

- Isolated nodes
- Degree of a node
- Connected graph
- Average degree of a graph
- Edge density: probability that any two nodes are connected

$$d = \frac{E}{(N * N - 1) / 2}$$

- Isolated node: v5
- Degree of a node:  
 $d(v1)=2, d(v4)=1$
- Average degree of a graph:  
 $D = (2+2+2+1+0)/5 = 1.4$
- Edge density  
 $d = 4 / (5 * 4 / 2) = 0.4$



# Examples: edge density

	nodes	edges	density [%]
Autobahnen	1 168	2 486	0.18
Internet	6 524	29 629	0.0696
www	325 729	1 497 135	0.0014
Power Grid	4 677	12 500	0.0572

sparse network  
(density ~ 1%)

metabolic	422	1 972	1.3
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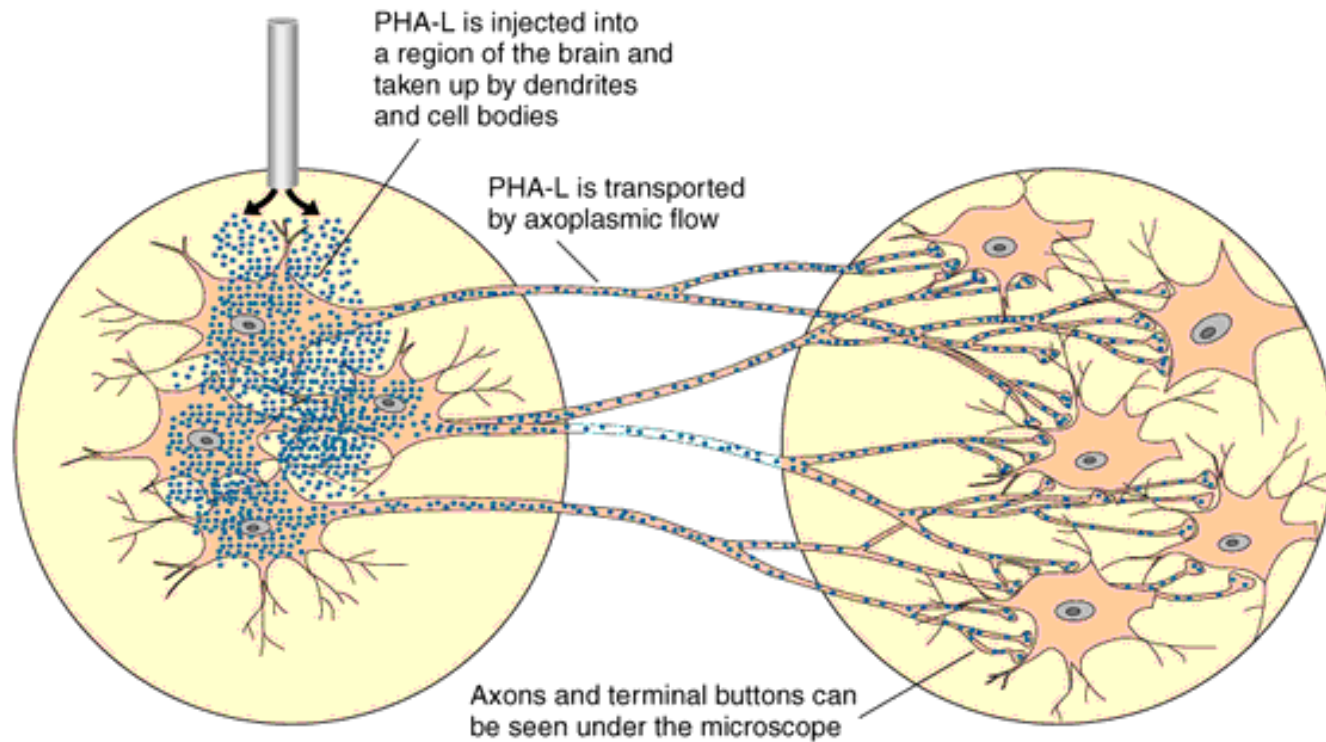
<i>C. Elegans</i>	202	2 540	6.3
(partial network)			
macaque	73	835	16

dense network  
(density > 5%)

How can the fibre tract network structure be examined?



# Tract tracing with dyes\*



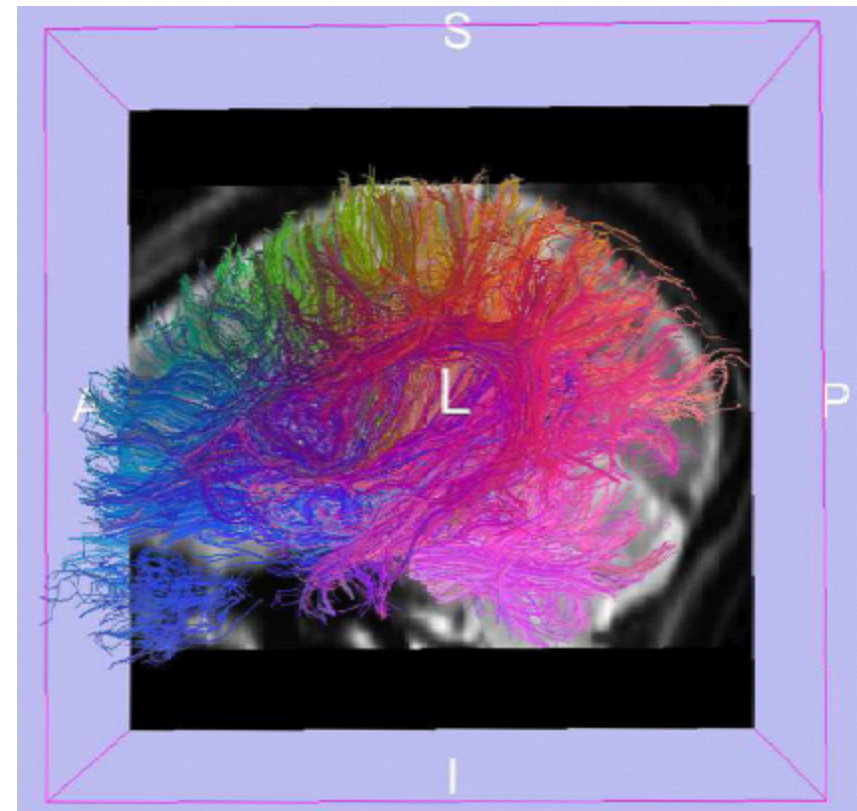
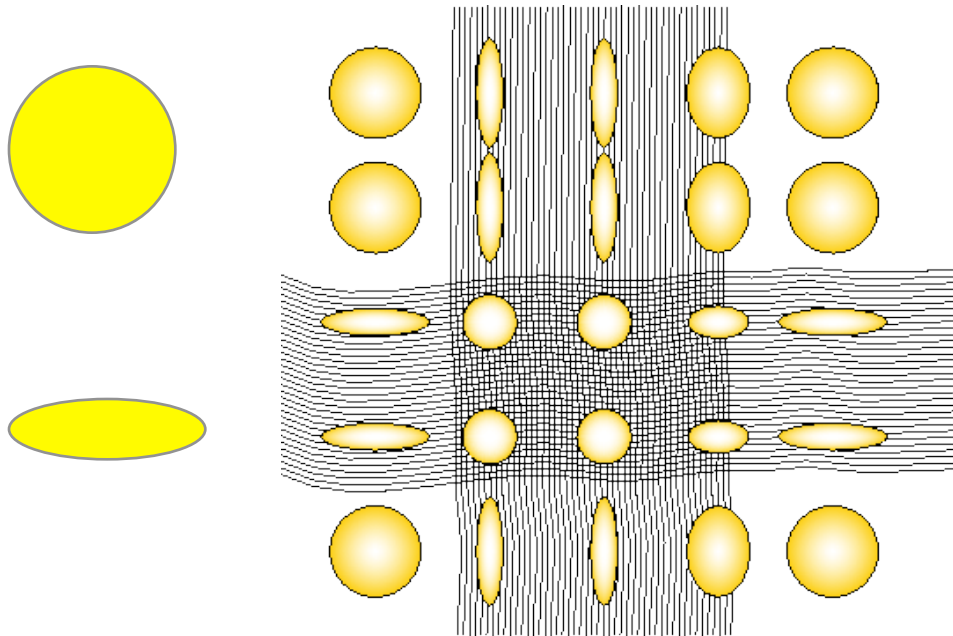
PHA-L: Phaseolus vulgaris-leucoagglutinin

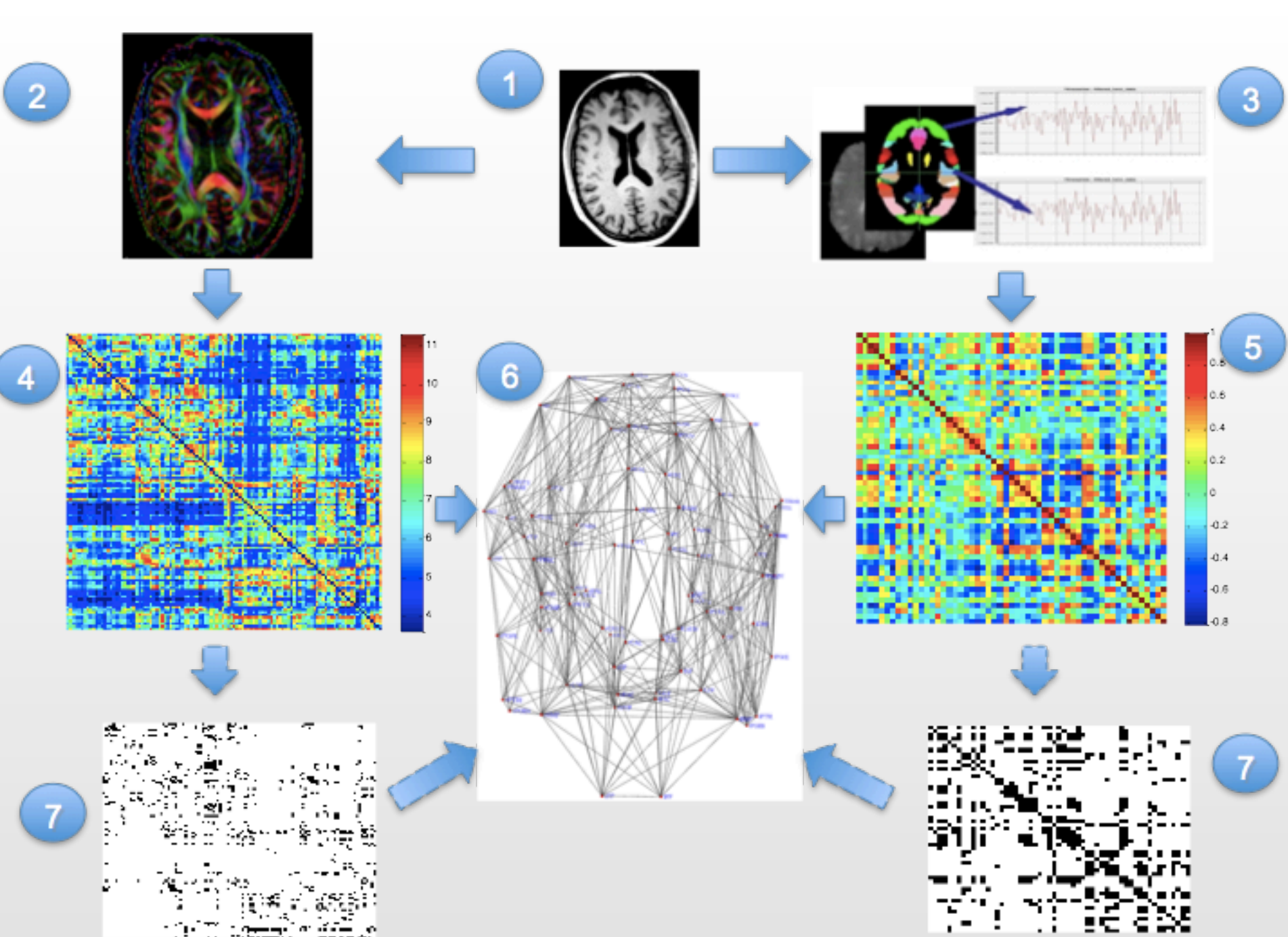
Anterograde:	soma → synapse
Retrograde:	soma ← synapse

\* Horseradish peroxidase (HRP) method; fluorescent microspheres; Phaseolus vulgaris-leucoagglutinin (PHA-L) method; Fluoro-Gold; Cholera B-toxin; Dil; tritiated amino acids

New  
Est. 1994

# Diffusion Tensor Imaging (DTI)

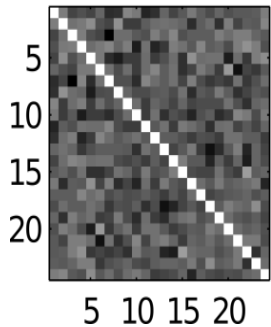
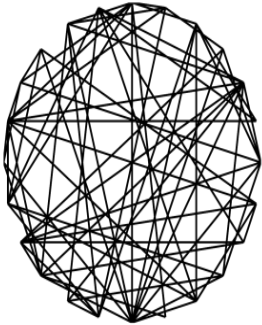




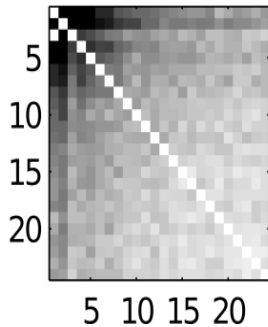
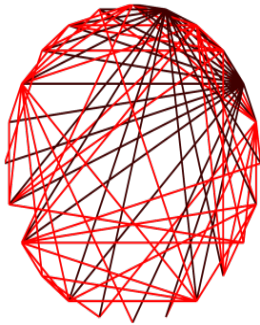
# Topological network organisation

# Archetypes of complex networks

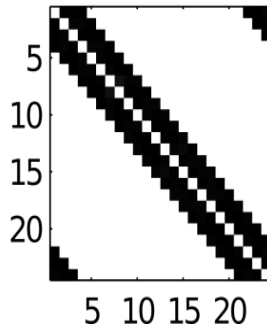
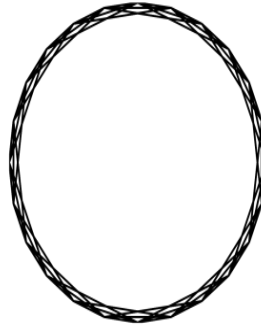
A Erdős-Rényi random



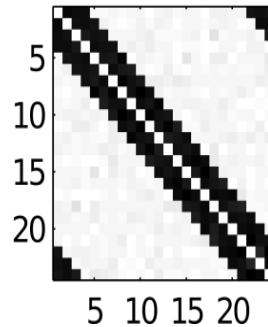
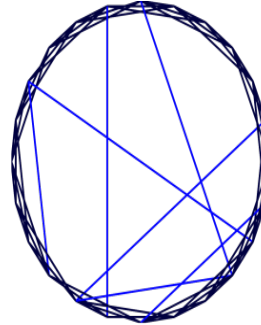
B Scale-free



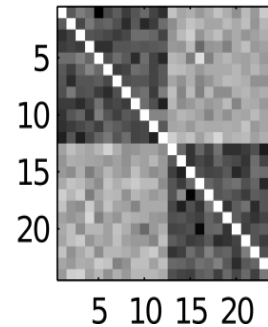
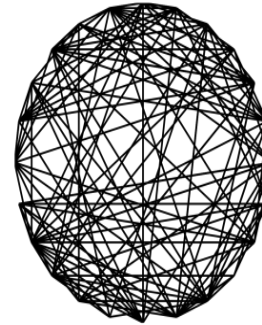
C Regular



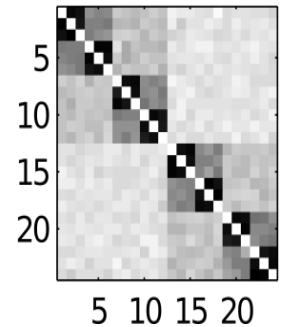
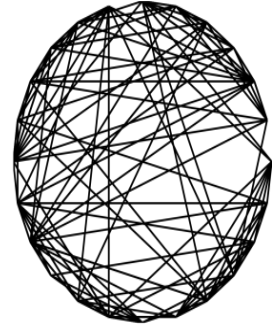
D Small-world



E Modular



F Hierarchical



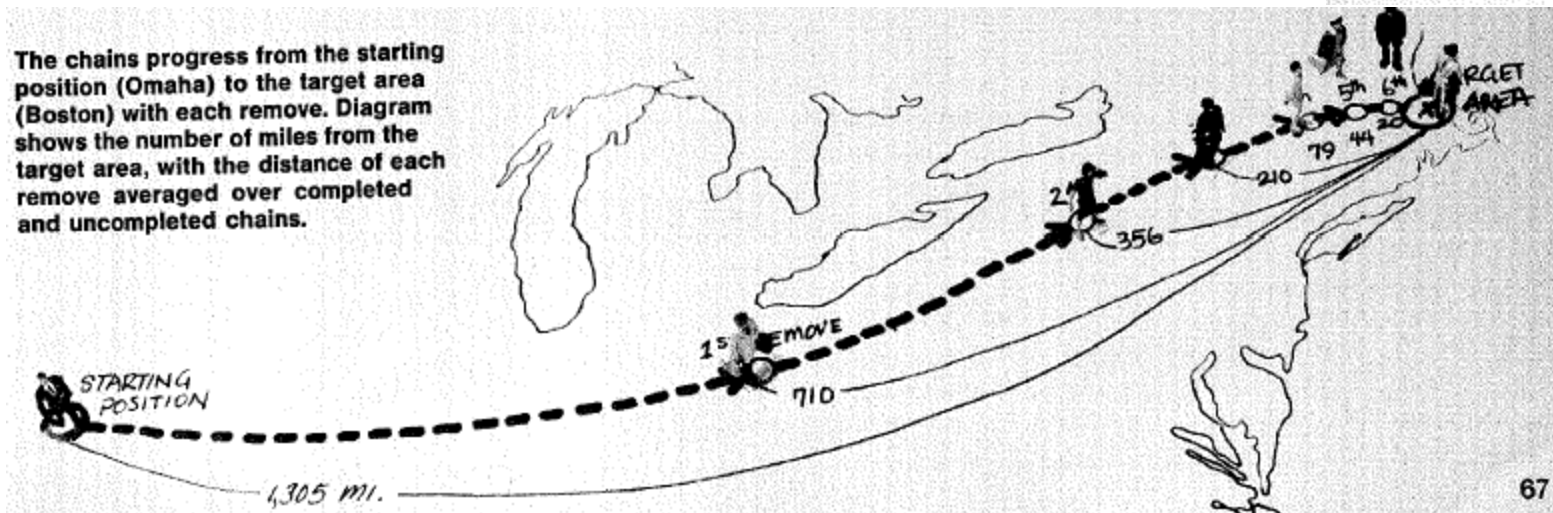
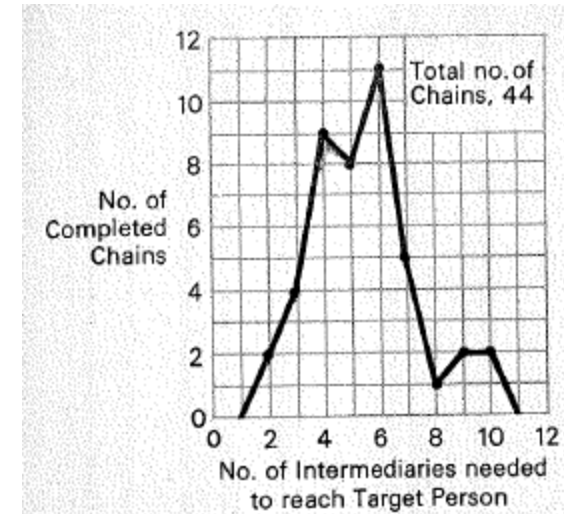
Note: real complex networks show a *combination* of these types!



# It's a small world

**Nodes:** individuals

**Links:** social relationship





Austin Powers



Robert Wagner



Let's make  
it legal



Wild Things



Kevin Bacon

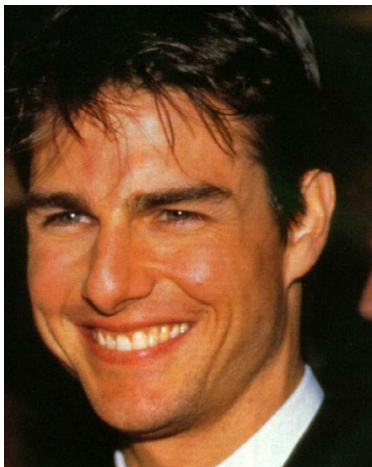
What Price Glory



Barry Norton



Monsieur  
Verdoux



A Few  
Good Man



# Network properties

## Clustering coefficient

Neighbours = nodes that are directly connected

local clustering coefficient

$C_{\text{local}}$  = average connectivity between neighbours

$C_{\text{local}}=1$  -> all neighbours are connected

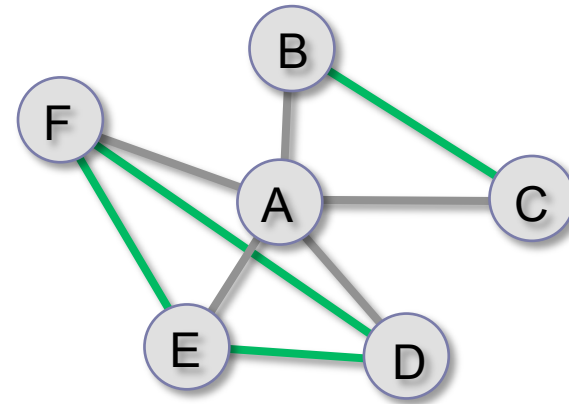
$C$  : global clustering coefficient (average over all nodes)

## Characteristic path length

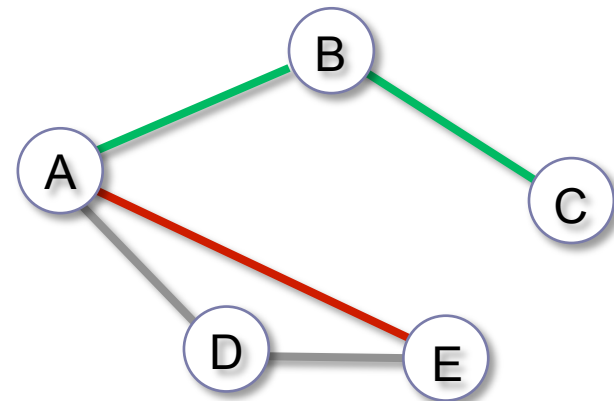
Shortest path between nodes  $i$  and  $j$ :

$L_{ij}$  = minimum number of connections to cross to go from one node to the other node

Characteristic path length  $L$  = average of shortest path lengths for all pairs of nodes



$$C_A = 4/10 = 0.4$$



Shortest path lengths:

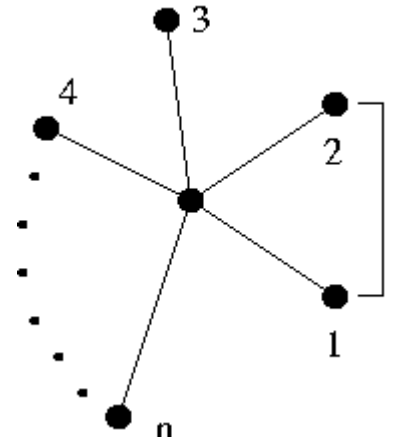
**A -> C** : 2

**A -> E** : 1



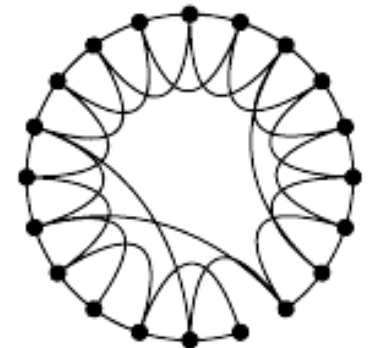
# Small-world networks

*Clustering coefficient* is higher than in random networks  
(e.g. 40% compared to 15% for the macaque monkey)



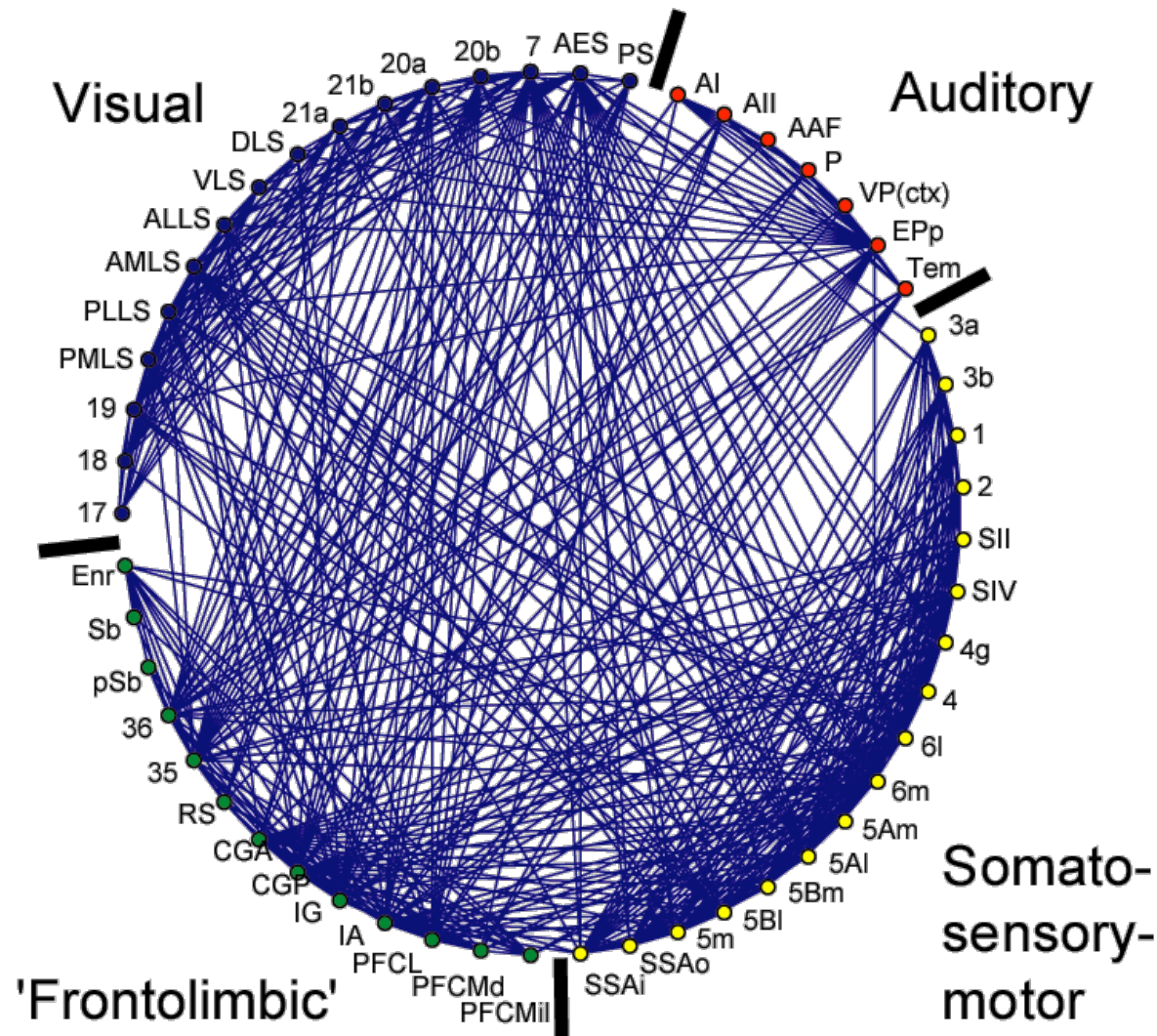
*Characteristic Path Length* is comparable to random networks

Small-world



Watts & Strogatz, Nature, 1998

# Modular small-world connectivity



## Small-world

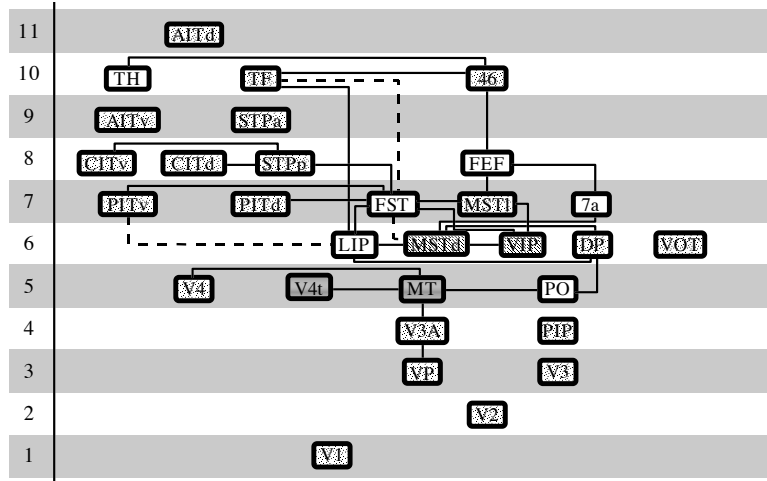
Neighbours are well connected; short characteristic path length ( $\sim 2$ )

## Modular

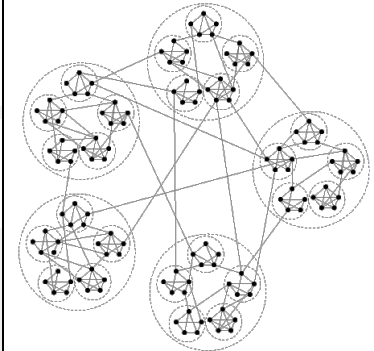
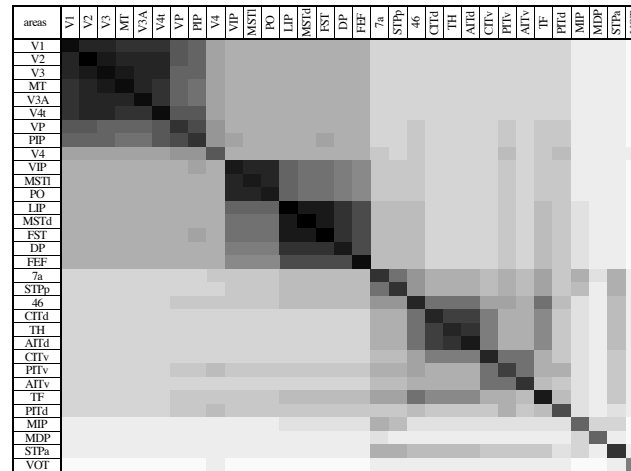
Clusters: relatively more connections within the cluster than between clusters

# Hierarchy

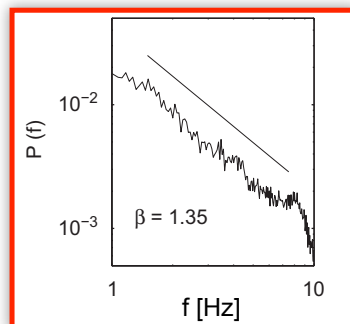
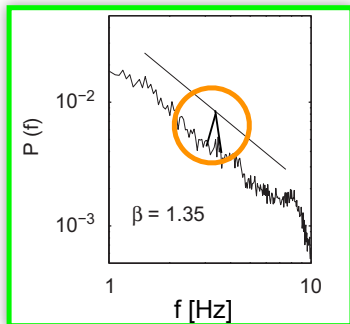
## Sequential



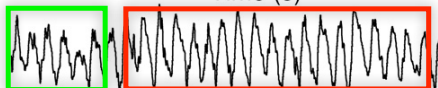
## Topological



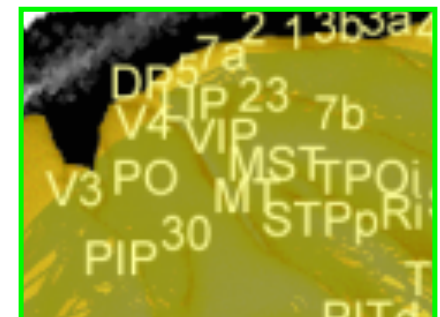
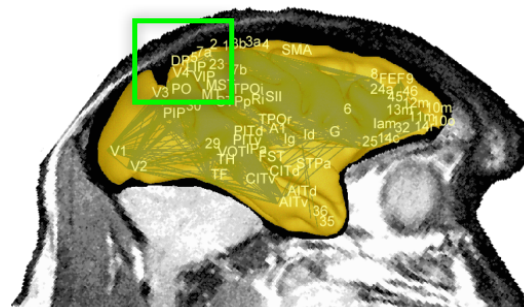
## Temporal



Time (s)



## Spatial



Kaiser et al. (2010) *Frontiers in Neuroinformatics*  
 Hilgetag & Kaiser *PLoS Comput. Biol.* (in preparation)

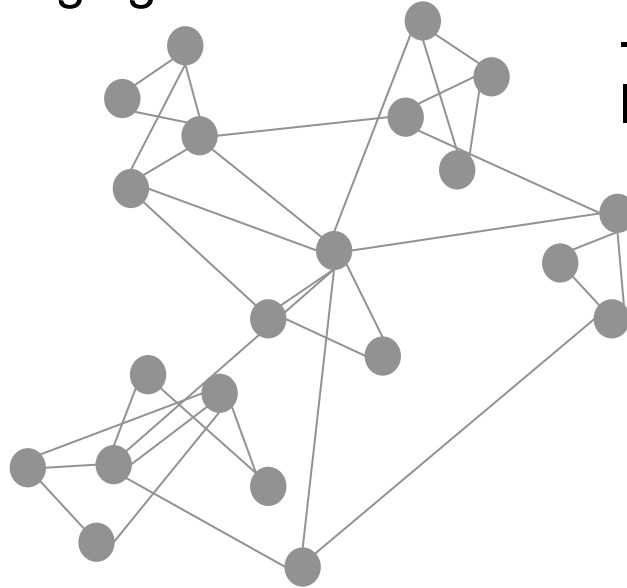
# Summary

## 2. Finding structural fibre tract connectivity:

- Diffusion tensor imaging
- Tract tracing

## 3. Topological properties:

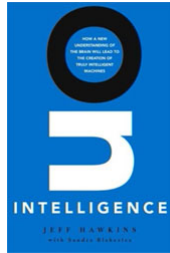
- multiple clusters/ modularity
- small-world: path lengths and local neighbourhood clustering



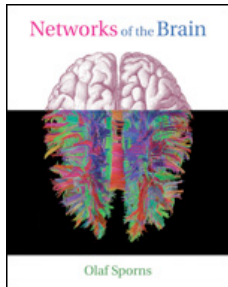
## 1. Types of connections:

- Structural
- Functional
- Effective

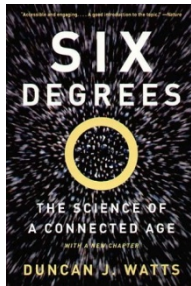
# Further readings



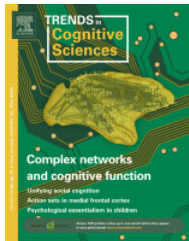
Jeff Hawkins with Sandra Blakeslee.  
*On Intelligence*. Henry Holt and Company, 2004



Olaf Sporns. *Networks of the Brain*. MIT Press, 2010



Duncan J. Watts. *Six Degrees: The Science of a Connected Age*. Norton & Company, 2004



Sporns, Chialvo, Kaiser, Hilgetag. Trends in Cognitive Sciences  
(September 2004) [www.biological-networks.org](http://www.biological-networks.org)