A Tutorial in Connectome Analysis (I):
Topological and Spatial Features of Brain Networks

Dr Marcus Kaiser

School of Computing Science / Institute of Neuroscience
Newcastle University
United Kingdom

WCU Dept of Brain & Cognitive Sciences
Seoul National University
South Korea
http://bcs.snu.ac.kr/

http://www.biological-networks.org
Outline

• What are neural networks?
• Introduction to network analysis
• How can the fibre tract network structure be examined?
• Topological network organisation
What are neural networks?
Levels of connectivity

Axons between neurons

Links between cortical columns

Fibre tracts between brain areas
Types of connectivity

- Structural / Anatomical (connection): two regions are connected by a fibre tract
- Functional (correlation): two regions are active at the same time
- Effective (causation): region A modulates activity in region B

Cortical networks

Dorsal and ventral visual pathway

Visual system
Introduction to network analysis
Network Science

Rapidly expanding field:
Barabasi & Albert, *Science* (October 1999) cited 4,000+ times

Modelling of SARS spreading over the airline network
(Hufnagel, *PNAS*, 2004)

Identity and Search in Social Networks
(Watts et al., *Science*, 2002)

The Large-Scale Organization of Metabolic Networks.
(Jeong et al., *Nature*, 2000)

First textbook on brain connectivity
(Sporns, ‘Networks of the Brain’, MIT Press, October 2010)
Origin of graph theory:
Leonhard Euler, 1736

Bridges over the river Pregel in Königsberg (now Kaliningrad)
Euler tour: path that visits each edge and returns to the origin
Graphs

- Graph: set of nodes and edges (non-directed)
  \[ G = (V,E) \]
- Set of nodes: \( V \) (singular: vertex; plural: vertices)
- Set of edges: \( E \subseteq V \times V \)
- E.g., \( V=\{v_1,v_2,v_3,v_4\} \),
  \[ E=\{(v_1,v_2), (v_1,v_3), (v_2,v_3), (v_3,v_4)\} \]

Topology
Directed graphs (Digraphs)

- Graph: set of nodes and *arcs* (directed)
- Set of nodes (vertices): $V$
- Set of edges: $E \subseteq V \times V$, the order matters
- E.g., $V=\{v_1,v_2,v_3,v_4\}$,
  $E=\{(v_1,v_2), (v_1,v_3), (v_2,v_3), (v_3,v_4), (v_4,v_1)\}$
Graphs and Networks

In theory (mathematics)
Graph: $G = (V, E)$

Network: $N = (G, s, t, c)$
defined by graph $G$ with source $s$, sink $t$, and edge capacity $c$
(examples: electricity/power grid, water flow, metabolic flux)

In reality (CS, engineering, economics, life and social sciences): term network used throughout (as in this course)
Nodes in graphs

- Isolated nodes
- Degree of a node
- Connected graph
- Average degree of a graph
- Edge density: probability that any two nodes are connected
  \[ d = \frac{\text{E}(N \times N - 1)}{2} \]

- Isolated node: v5
- Degree of a node:
  \[ d(v1) = 2, \quad d(v4) = 1 \]
- Average degree of a graph:
  \[ D = \frac{2+2+2+1+0}{5} = 1.4 \]
- Edge density
  \[ d = \frac{4}{5 \times 4 / 2} = 0.4 \]
Examples: edge density

<table>
<thead>
<tr>
<th></th>
<th>nodes</th>
<th>edges</th>
<th>density [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autobahnen</td>
<td>1168</td>
<td>2486</td>
<td>0.18</td>
</tr>
<tr>
<td>Internet</td>
<td>6524</td>
<td>29629</td>
<td>0.0696</td>
</tr>
<tr>
<td>www</td>
<td>325729</td>
<td>1497135</td>
<td>0.0014</td>
</tr>
<tr>
<td>Power Grid</td>
<td>4677</td>
<td>12500</td>
<td>0.0572</td>
</tr>
<tr>
<td>metabolic</td>
<td>422</td>
<td>1972</td>
<td>1.3</td>
</tr>
<tr>
<td><em>C. Elegans</em></td>
<td>202</td>
<td>2540</td>
<td>6.3</td>
</tr>
<tr>
<td>(partial network)</td>
<td>73</td>
<td>835</td>
<td>16</td>
</tr>
</tbody>
</table>

sparse network (density ~ 1%)

dense network (density > 5%)
How can the fibre tract network structure be examined?
Tract tracing with dyes*

Anterograde: soma → synapse
Retrograde: soma ← synapse

* Horseradish peroxidase (HRP) method; fluorescent microspheres; Phaseolus vulgaris-leucoagglutinin (PHA-L) method; Fluoro-Gold; Cholera B-toxin; Dil; tritiated amino acids
Diffusion Tensor Imaging (DTI)
Topological network organisation
Archetypes of complex networks

A Erdős-Rényi random  B Scale-free  C Regular  D Small-world  E Modular  F Hierarchical

Note: real complex networks show a combination of these types!

Kaiser (2011) Neuroimage
It’s a small world

**Nodes**: individuals

**Links**: social relationship

The chains progress from the starting position (Omaha) to the target area (Boston) with each remove. Diagram shows the number of miles from the target area, with the distance of each remove averaged over completed and uncompleted chains.

Austin Powers

A Few Good Man

Wild Things

Let's make it legal

Robert Wagner

Kevin Bacon

Barry Norton

What Price Glory

Monsieur Verdoux
Network properties

Clustering coefficient
Neighbours = nodes that are directly connected

Local clustering coefficient
$C_{local} = \text{average connectivity between neighbours}$
$C_{local} = 1 \rightarrow \text{all neighbours are connected}$

$C$: global clustering coefficient (average over all nodes)

Characteristic path length
Shortest path between nodes $i$ and $j$:
$L_{ij} = \text{minimum number of connections to cross to go from one node to the other node}$

Characteristic path length $L = \text{average of shortest path lengths for all pairs of nodes}$

$C_A = \frac{4}{10} = 0.4$

Shortest path lengths:
$A \rightarrow C : 2$
$A \rightarrow E : 1$
**Small-world networks**

*Clustering coefficient* is higher than in random networks

(e.g. 40% compared to 15% for the macaque monkey)

*Characteristic Path Length* is comparable to random networks

Modular small-world connectivity

Small-world
Neighbours are well connected; short characteristic path length (~2)

Modular
Clusters: relatively more connections within the cluster than between clusters

Kaiser et al. (2010) Frontiers in Neuroinformatics
Summary

1. Types of connections:
   - Structural
   - Functional
   - Effective

2. Finding structural fibre tract connectivity:
   - Diffusion tensor imaging
   - Tract tracing

3. Topological properties:
   - multiple clusters/ modularity
   - small-world: path lengths and local neighbourhood clustering
Further readings

Jeff Hawkins with Sandra Blakeslee. On Intelligence. Henry Holt and Company, 2004

Olaf Sporns. Networks of the Brain. MIT Press, 2010
